

[EIGHT PROBLEMS](#) / [WORKED ANSWERS](#) / [A PRACTICE PLAN](#)

# A model before a number

## Eight problems to find your next practice session

Set aside about 30 minutes as a suggested practice window, not a measured benchmark. Work without the answer pages first. For each problem, write the model, your calculation and one check. If you get stuck, record where. These original educational exercises are not employer interview questions or a hiring assessment.

Use paper or a calculator. An exact fraction is welcome; a justified decimal is equally useful. Do not award yourself credit just for recognizing an answer after reading it. Explain the reasoning again with the solution covered. The final page turns mistakes into a practice plan; it does not estimate your chance of getting hired.

## How to use these pages

Questions are on pages 2-5. Worked answers are on pages 6-9. Keep those pages covered until you have recorded your own reasoning. Page 10 helps you select your next practice session.

[Edition 01](#) / [Editorial working copy](#)

This is a small diagnostic of stated reasoning tasks. It does not cover the full scope of quant recruiting, advanced statistics, algorithms, stochastic calculus or role-specific technical interviews. All eight scenarios are original teaching examples; none is presented as an actual employer question.

TRY FIRST / KEEP THE ANSWERS COVERED

## 01 / The box you did not see

### CONDITIONING

Choose box A with probability  $1/3$  and box B with probability  $2/3$ . Box A contains three blue tokens and one white token. Box B contains one blue token and three white tokens. Draw one token uniformly from the chosen box. It is blue. What is the probability you chose box A?

The box choice happens first. Tokens are equally likely within each box. You observe the color, not the box.

MODEL / CALCULATION / CHECK

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## 02 / Count what appeared

### EXPECTATION

A generator independently returns one of four labels, each with probability  $1/4$ , on every call. You make three calls. What is the expected number of different labels observed?

Calls are independent and identically distributed. A label counts once even if it appears repeatedly.

MODEL / CALCULATION / CHECK

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TRY FIRST / KEEP THE ANSWERS COVERED

## 03 / Two copies of one risk

### DEPENDENCE

$X$  is equally likely to be  $+1$  or  $-1$ . Portfolio A pays  $2X$ . Portfolio B pays  $X + Y$ , where  $Y$  is an independent copy of  $X$ . Find the expectation and variance of each payoff. Which payoff has the greater probability of being strictly positive?

These are stipulated unitless payoffs. Strictly positive excludes zero. No investment recommendation is implied.

MODEL / CALCULATION / CHECK

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## 04 / Pay for one more draw?

### STOPPING

A draw returns either 1 or 9, each with probability  $1/2$ . After seeing the first draw, you can keep it or discard it and pay a cost of 2 for a second independent draw. If you redraw, you must accept the second value minus the cost. What is the optimal rule for maximizing expected net payoff, and what is its value before the first draw?

There are at most two draws. The cost is paid only if you redraw. You cannot recover the first draw afterward.

MODEL / CALCULATION / CHECK

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TRY FIRST / KEEP THE ANSWERS COVERED

## 05 / Only the excess is paid

CONTINUOUS PROBABILITY

$U$  is uniformly distributed on the interval from 0 to 10. A ticket pays  $\max(U - 6, 0)$ . Find the probability of a positive payment and the expected payment.

The uniform density is  $1/10$  over the interval. The payoff is the excess above six, not the entire value of  $U$ .

MODEL / CALCULATION / CHECK

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## 06 / What does more data change?

ESTIMATION

Independent observations follow a normal distribution with unknown mean and known standard deviation 8. You observe 64 values and obtain sample mean 3. Give a two-sided 95% confidence interval for the population mean using 1.96. If the sample size becomes 256 under the same model, how does the interval's width change?

The standard deviation is known, observations are independent and the stated normal model is correct. For the width comparison, hold the standard deviation fixed.

MODEL / CALCULATION / CHECK

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TRY FIRST / KEEP THE ANSWERS COVERED

## 07 / The trade changes the evidence

### INFORMATION

A toy asset's terminal value  $V$  is equally likely to be 8 or 12. You quote a bid of 9 and an ask of 11 for one unit. With probability  $1/2$ , the arriving counterparty knows  $V$  and buys only when  $V$  is 12 or sells only when  $V$  is 8. Otherwise the counterparty chooses buy or sell with equal probability, independently of  $V$ . What is your expected profit per arrival? After a buy, what is the probability  $V$  is 12?

Counterparty type is independent of  $V$ . Exactly one trade occurs at the posted quote. Ignore fees, inventory costs and later trading; settlement is at  $V$ .

MODEL / CALCULATION / CHECK

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## 08 / A final segment used twice

### EVALUATION

You fit three forecasting procedures on records 1 through 600. You compare all three on records 601 through 900 and select the one with the lowest error. You report that same selected error as an untouched estimate of the winning procedure's future performance. What is wrong with the description, and what would improve the evaluation?

Records are chronologically ordered. All features are genuinely available at their prediction times. The issue here is model selection, not a hidden future feature.

MODEL / CALCULATION / CHECK

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WORKED ANSWERS / CHECK YOUR REASONING

## 01 / The box you did not see

**$3/5 = 0.60$**

The joint probability of A and blue is  $(1/3)(3/4) = 1/4$ . The joint probability of B and blue is  $(2/3)(1/4) = 1/6$ . Blue therefore has probability  $1/4 + 1/6 = 5/12$ . Conditioning divides the A-and-blue mass by all blue mass:  $(1/4)/(5/12) = 3/5$ . As a count check, imagine twelve equally weighted outcomes: three A-blue outcomes, one A-white outcome, two B-blue outcomes and six B-white outcomes. Three of the five blue outcomes came from A.

**Watch for:** Answering  $3/4$  ignores that box B was twice as likely to be selected. Pooling the physical tokens also discards the box-selection probabilities.

**Change one assumption:** If the box probabilities become equal, the answer becomes  $3/4$ . Explain why that changed without changing the contents of either box.

## 02 / Count what appeared

**$37/16 = 2.3125$  distinct labels**

For each label, define an indicator equal to one if it appears at least once. Its probability of appearing is  $1 - (3/4)^3 = 37/64$ . The number of distinct labels is the sum of the four indicators, so its expectation is  $4(37/64) = 37/16$ . The indicators are not independent, but linearity of expectation does not require that. Independence between calls was used when computing the absence probability. The answer lies between one and three, the smallest and largest possible distinct counts.

**Watch for:** Three calls do not imply three different labels. Avoid multiplying independent-looking probabilities without naming which events are independent.

**Change one assumption:** With two calls the expectation is  $7/4$ . Derive that both with indicators and by separating repeated from different labels.

WORKED ANSWERS / CHECK YOUR REASONING

### 03 / Two copies of one risk

**Both means are 0. Variance:  $A = 4$ ;  $B = 2$ . Positive probability:  $A = 1/2$ ;  $B = 1/4$ .**

For A, the only payoffs are +2 and -2, each with probability  $1/2$ . The mean is zero and the expected squared payoff is four, so the variance is four. For B, payoffs +2, 0 and -2 have probabilities  $1/4$ ,  $1/2$  and  $1/4$ . Its mean is zero and its expected squared payoff is two. Equivalently, independent X and Y each have variance one and covariance zero. A is positive half the time; B is positive only when both independent signs are positive. B has lower variance, but that alone does not make its probability of a strictly positive payoff larger.

**Watch for:** Treating  $X + X$  as two independent draws loses the covariance term. Comparing positive probability alone also ignores the sizes and probabilities of losses and zeros.

**Change one assumption:** Replace Y by -X. The sum is always zero, its variance is zero and its probability of being strictly positive is zero.

### 04 / Pay for one more draw?

**Redraw after 1; keep 9. Expected net payoff = 6.**

The redraw option has expected net payoff  $(1 + 9)/2 - 2 = 3$ . Compare the observed first value with three: replacing a one improves its conditional expectation; replacing a nine worsens it. Before observing anything, the optimized expected payoff is  $(1/2)(3) + (1/2)(9) = 6$ . An outcome-tree check gives net payoffs -1 with probability  $1/4$ , 7 with probability  $1/4$  and 9 with probability  $1/2$ . Their weighted sum is six. A rule can maximize expectation while still producing a negative payoff on some paths.

**Watch for:** Averaging the best of two draws describes a different game in which you can recall the first value. Here the first value is discarded before the second is revealed.

**Change one assumption:** If the redraw cost is 4, replacing a one and keeping it have the same expectation. If the cost exceeds 4, keeping either first value is optimal.

## WORKED ANSWERS / CHECK YOUR REASONING

## 05 / Only the excess is paid

**Positive probability =  $2/5$ . Expected payment =  $4/5 = 0.8$ .**

The payment is positive for  $U$  greater than six, a region of length four out of ten, so its probability is 0.4. Conditional on that region,  $U$  is uniform from six to ten and the excess has mean two. Total expectation gives  $0.4 \times 2 = 0.8$ . The same result follows from integrating  $(u - 6)/10$  from six to ten: the antiderivative is  $(u - 6)^2/20$  and the upper endpoint gives  $16/20$ . A bound check is  $0 \leq \text{expected payment} \leq 4 \times 0.4 = 1.6$ , because four is the maximum payment.

**Watch for:** Multiplying 0.4 by the unconditional mean of  $U$  uses the wrong conditional value and the wrong payoff. A probability and an expected payment also have different units.

**Change one assumption:** If the threshold becomes eight, the positive probability is  $1/5$  and the expected payment is  $1/5$ . Explain why the payment falls more sharply than the positive probability.

## 06 / What does more data change?

**[1.04, 4.96]. Quadrupling the sample size halves the width.**

The standard error is  $8/\sqrt{64} = 1$ . The interval is 3 plus or minus 1.96, giving endpoints 1.04 and 4.96 and total width 3.92. At 256 observations the standard error is  $8/16 = 0.5$ , so the width becomes 1.96. The new interval's center would depend on its new sample mean; only its width is determined here. Under repeated independent samples satisfying the model, about 95% of intervals from this procedure cover the fixed true mean. This does not say that 95% of individual observations fall inside the displayed interval.

**Watch for:** Dividing standard deviation by  $n$  instead of  $\sqrt{n}$  overstates precision. Extra correlated observations also need not provide the information assumed by the independent model.

**Change one assumption:** If standard deviation doubles while sample size stays at 64, the width doubles. Name the modeling assumption you would inspect before trusting a narrow interval from a time series.

WORKED ANSWERS / CHECK YOUR REASONING

## 07 / The trade changes the evidence

**Expected profit = 0. After a buy,  $P(V = 12) = 3/4$ .**

Against an informed counterparty you always lose one: selling at eleven when value is twelve or buying at nine when value is eight. Against a random counterparty the expected profit is one for either side, since expected value is ten. Mixing the two types equally gives zero expected profit. A buy has probability one half. The joint probability of a buy and  $V = 12$  is  $(1/2)(1/2)$  for an informed buyer plus  $(1/2)(1/2)(1/2)$  for a random buyer, totaling  $3/8$ . Divide by  $1/2$  to get  $3/4$ . Conditional expected value after a buy is 11, exactly the ask.

**Watch for:** A spread around the unconditional mean does not ensure positive expected profit. The direction of the trade can reveal information about value.

**Change one assumption:** If the informed share becomes  $3/4$  with quotes unchanged, expected profit becomes  $-1/2$  per arrival. Keep the calculation conditional on counterparty type.

## 08 / A final segment used twice

**The segment was used for selection. It is not untouched final evaluation.**

Training coefficients on earlier data addresses one boundary, but choosing the procedure from records 601 through 900 uses those outcomes too. The observed best error can benefit from selection noise. Call this segment validation data and preserve the result as exploratory evidence. Freeze the selected procedure and evaluate it on a later, genuinely unused segment, or design a chronological nested evaluation that repeats the complete selection process within its permitted history. Neither step guarantees future performance: distribution change, dependence, sample size and implementation still matter. If you keep changing the procedure after inspecting the new segment, that segment also becomes part of development.

**Watch for:** The absence of future features does not prove a clean experiment. Researcher choices can use information from the evaluation set even when model-fitting code does not.

**Change one assumption:** If no new data is available, report the selection process and its limitation. Do not relabel the already viewed segment as untouched or claim the uncertainty has disappeared.

[NEXT SESSION](#) / [ONE REPAIR AT A TIME](#)

# Choose the next useful problem

**01.** For each question, record whether you stated the model, completed the calculation and performed a meaningful check. Use yes, partial or not yet for each; there is no hiring cutoff.

**02.** Choose one error type: setup, conditioning, algebra, dependence, stopping rule, interpretation or evaluation design. Write the exact line where your reasoning went wrong.

**03.** Read the matching free lesson when the site launches for one weak area. Then cover the answer and redo the problem. Finally solve its follow-up, naming what changed and what stayed fixed.

**04.** On a later practice session, explain the solution aloud from a blank page. Record whether the repair survived without recognizing the old wording. Buy more material only if you need a broader sequence of practice.

## Free reading paths

The planned website lessons below match the error each question exposes. The website is not yet launched; all worked answers needed for this diagnostic are included here.

01 / Condition on the full observed group

02 / Use one indicator per counted object

03 / Separate a dependence claim from a decision

04 / Work backward from the last allowed action

05 / Check a result against its possible range

06 / Name the estimator and its assumptions

07 / Separate an unconditional price from informed flow

08 / Audit decisions as well as features

## When a longer workbook helps

Use the Probability & Trading Interview Workbook for a connected sequence of finite probability, expectation, stopping, uncertainty and toy-market exercises. Use the Quant Research Project Lab for a reproducible synthetic experiment and evaluation practice. Inspect each book's free preview to judge its scope before purchase. Current editions are editorial drafts and are not for sale.